7.5 Roots and Zeros

Fundamental Theorem of Algebra

- all polynomial equations with a degree greater than zero have at least one root in the set of complex numbers
- a polynomial equation of the form P(x) = 0 of degree n with complex coefficients, has exactly n roots

How many roots?

ex:

$a - 10 = 0$ 1

$x^2 + 2x - 48 = 0$
2

$3a^3 + 18a = 0$
3

$y^5 - y = 0$
5

Solve # 3.

$3a^3 + 18a = 0$

$3a(a^2 + 6) = 0$

$a = 0 \quad a = \pm i\sqrt{6}$

1 $\mathbb{R}$ 2 imaginary

Solve # 4.

$y(y^4 - 1) = 0$

$y(y^2 + 1)(y^2 - 1) = 0$

$y^2 = -1 \quad y^2 = 1$

$y = 0 \quad y = \pm i \quad y = \pm 1$

3 $\mathbb{R}$ 2 imag.

Complex Conjugates
a, b $\in$ R b$\neq$0
If a + bi is a zero, then a - bi is a zero

1. If 3 + 2i is a zero, then __3 – 2i___ is also a zero.

2. If 5 - 3i is a zero, then 5+3i is also a zero.

3. If 1-√5 is a zero, then 1+√5 is also a zero.
(It also works for radicals)

4. If 6i is a zero, then −6i is also a zero.

Write a function with the following zeros.

3, 2 + i , 2 − i

Write a function with the following zeros.

-2, $2-\sqrt{3}$, $2+\sqrt{3}$

Sum
$2+\sqrt{3} + 2-\sqrt{3} = 4$

Product
$(2+\sqrt{3})(2-\sqrt{3}) = 1$

4 −3
$(x^2-4x+1)(x+2)=0$

x^3-4x^2+x
$+2x^2-8x+2$

$\boxed{x^3-2x^2-7x+2=0}$

Write a function with the following zeros.

-8, 5i

HW
p375-376
13, 15, 17,
35, 38, 39, 41